Postdoctoral Position

Stability and Control of Complex Dynamical Systems

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Location:	Laboratory of Analysis and Control of PDEs (UR13ES64)
	Department of Mathematics, Faculty of Sciences of Monastir
Salary:	1300 TND.
Starting date:	There is flexibility for the starting date.
Duration:	12 months with possible renewal for 12 additional months.
Required qualification:	Applicants should have a PhD in Mathematics (Control and Stability Theory, Dynamical Systems, Partial Differential Equations).
General topics:	Partial Differential Equations, Control and Stability Theory, Spectral Analysis, Fluid Mechanics, Stokes Equations.

Scientific Context

In the last decades there has been an increasing interest devoted to the study of the dynamics and control of partial differential equations (PDE). In the context of control theory, these topics are also often referred to as Distributed Parameter Systems (DPS). The mathematical formulation of the boundary control problem can be briefly summarized as follows: consider an evolution system described in terms of partial differential equations on which we are allowed to act by means of a suitable choice of the control on the right-hand side (resp. left-hand side) of the system. Given a time interval $0 < t < T$, and initial and final states, the goal is to determine whether there exists a control driving the given initial data to the given final ones in time T. More precisely, a control system can be written as

$$\begin{cases} y_t = \mathscr{A} y + \mathscr{B} u(t), \ 0 < t < T, \\ y(0) = y^0, \end{cases} \tag{1}$$

where y represents the state variables to be controlled. Here, y and u are lives, respectively, in a state space $\mathscr{H}$ and in a control space $\mathscr{U}$ which are in infinite dimensional spaces. In the above system, $\mathscr{A}$, $\mathscr{B}$ are linear operators such that $\mathscr{A} : \mathscr{D}(\mathscr{A}) \subset \mathscr{H} \to \mathscr{H}$ and $\mathscr{B} : \mathscr{U} \to \mathscr{D}(\mathscr{A}^*)'$. The linear operator $\mathscr{A}$ determines the equation that must be satisfied by the state variable y, according to the laws of physics and $\mathscr{B}$ is a boundary trace operator. To answer this question, we need a precise analysis of the adjoint homogeneous system to the system one wants to control. Roughly speaking, the control problem is shown to be equivalent to the uniqueness of solutions of the adjoint system, together with establishing suitable observability inequalities. Especially for one dimensional control systems, the moments method plays a key role to investigate exact observability results. This technique is essentially based on non-harmonic Fourier analysis and on some Ingham's inequalities [11]. The book of Ammari and Nicaise [2] gives the state of the art on this method.

Research Project

The main objective is to study the dynamics and control of some models of interconnected beams and wave equations, with the added possible presence of point masses at the joints. This is a classical problem in control engineering for its many applications in multi-body and multi-linked structures and its vibrations. To mention some examples, see [1, 3, 5] for networks of strings with attached masses, see [4, 10] for a Schrödinger equation with internal point masses, and [6, 7, 8, 9] for interconnected beams with attached masses.

Goals and Objectives For these kind of problems, the following issues may be addressed:

(1) The control problems for systems of elastic strings and beams.

(2) Problems of dynamics and stability of some quantum hybrid models.

(3) The analyticity of the semi-groupe generated by the Stokes operator.

(4) The spectral analysis of interconnected beams with attached masses.

Planning The postdoctoral stay could be scheduled as follows:

Step 1 **::** Bibliographical study of stability theory [2]. In particular, the weighted observability inequality, the frequency domain method and the theory of Riesz basis.

Step 2 **::** Bibliographical study of control theory [11]. In particular, the theory of nonharmonic Fourier series, Ingham's type inequalities and the Hilbert uniqueness method of Lions.

Step 3 **::** Bibliographical study of semi-groupe theory [1, 3].

Step 4 **::** Proof of some exact controllability and stability results.

N.B: This program will be adapted according to the experience of the candidate. Interested person may contact and send by email a CV to Professor Kaïs Ammari.

References

[1] K. Ammari and C. Amrouche, Resolvent estimates for wave operators in Lipschitz domains, *Calcul. of Variat. and Part. Diff. Eqs.,* **60** (2021), 1–34.

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[3] K. Ammari and F. Shel, Stability of Elastic Multi-Link Structures, SpringerBriefs in Mathematics, Springer Nature Switzerland AG, (2022).

[4] K. Ammari, D. Mercier and V. Régnier, Spectral analysis of the Schrödinger operator on binary tree-shaped networks and applications, *J. Differential Equations,* **259** (2015), 6923–6959.

[5] K. Ammari, D. Mercier, Boundary feedback stabilization of a chain of serially connected strings, *Evol. Equ. Control Theory.,* **4** (2015), 1–19.

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[9] C. Castro and E. Zuazua, *Exact boundary controllability of two Euler-Bernoulli beams connected by a point mass*, *Math. Comput. Mod.,* **32** (2000), 955–969.

[10] S. Hansen, Exact boundary controllability of a Schrödinger equation with an internal point mass, *Amer. Cont. Conf. May 24–26, Seattle, USA, (2017)*.

[11] V. Komornik and P. Loreti, Fourier series in control theory, Springer Science & Business Media, (2005).